MATH 2230 HW 5

① It follows from z^n having antiderivative in $\mathbb{C}$. $\forall n \in \mathbb{N}$

2a $\int_0^{1+i} z^2\,dz = \left[\frac{z^3}{3}\right]_0^{1+i} = \frac{2}{3}(-1+i)$

2b $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz = \left[2\sin\left(\frac{z}{2}\right)\right]_0^{\pi+2i} = 2\left(\frac{e^{\frac{i\pi-2}{2}} - e^{\frac{-i\pi+2}{2}}}{2i} - 0\right)$

$= e + e^{-1}$

2c $\int_1^3 (z-2)^3\,dz = \left[\frac{1}{4}(z-2)^4\right]_1^3 = 0.$

③ It follows from $(z-z_0)^{n-1}$ having antiderivative in $\mathbb{C}$ $\forall n = \pm1, \pm2 \ldots$

④ Since $z^{1/2}$ is analytic in the lower half plane (simply connected) if we choose the branch to be $\{\pi/2 < \arg z < 5\pi/2\}$.

Its antiderivative is $\frac{2}{3}z^{3/2}$ which is analytic in that domain.

$\int_{-3}^{3} z^{1/2}\,dz = \left[\frac{2}{3}z^{3/2}\right]_{-3}^{3} = 2\sqrt{3}(-1+i)$

⑤ Since z^i is analytic in the upper half plane (simply connected) with the chosen branch. Its antiderivative is $\frac{1}{i+1}z^{i+1}$ in that domain.

$\int_{-1}^{1} z^i\,dz = \left[\frac{1}{i+1}z^{i+1}\right]_{-1}^{1} = \frac{1+e^{-\pi}}{2}(1-i).$